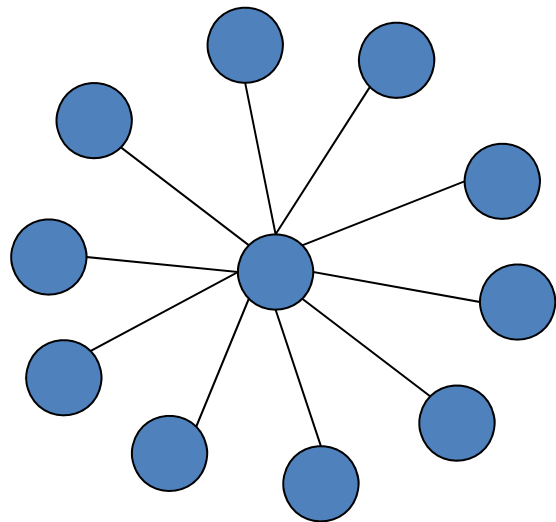


Strategic Network Formation



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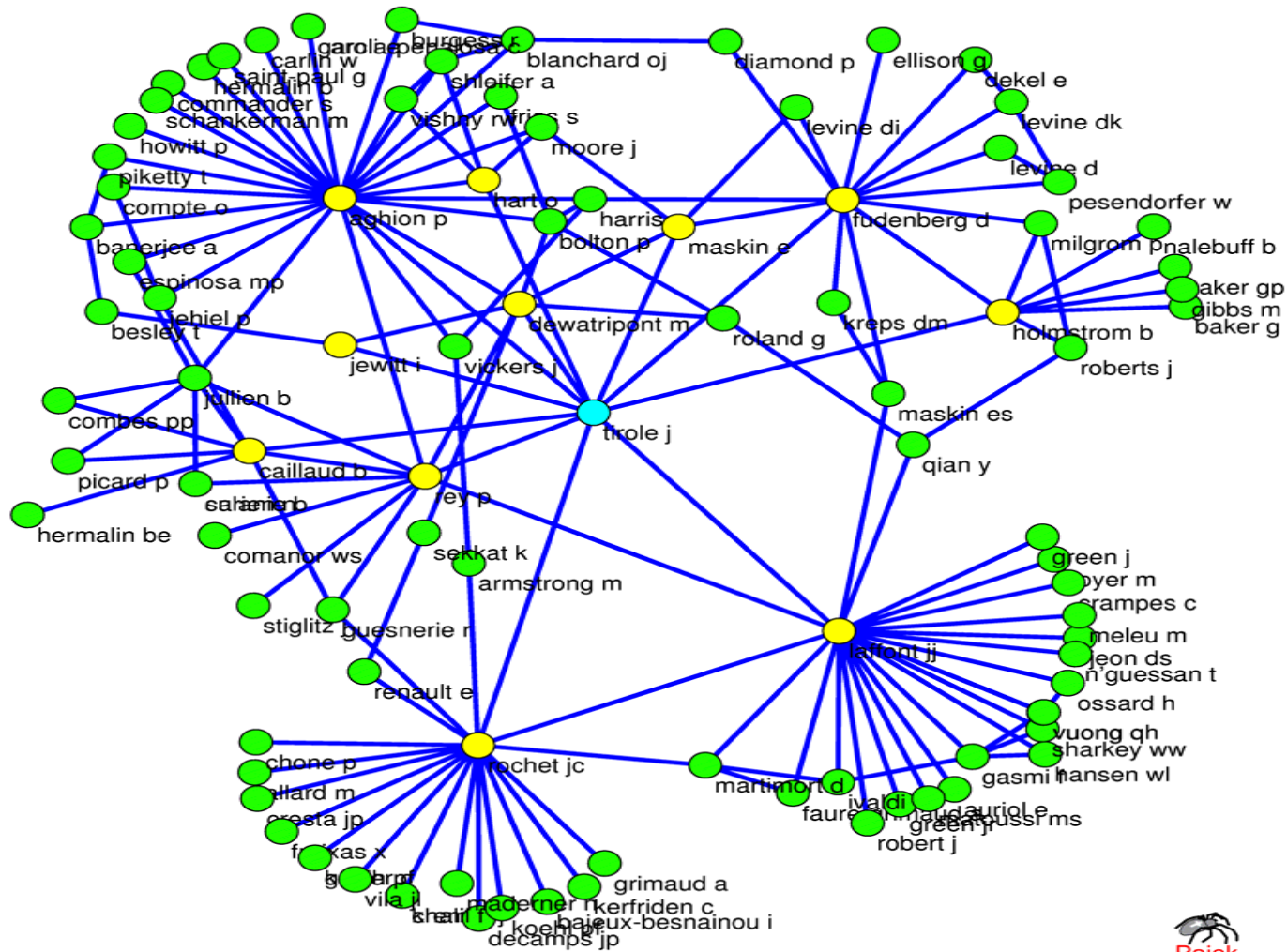
Outline

1. Structure of networks
2. Strategic foundations of networks
3. Theory 1: Communication networks
 - Stars, hubs and spokes
 - Networks Advantages
 - Good and bad networks
4. Theory 2: Networks and markets
 - Collaborating to compete
 - Dominant groups
 - Subsidizing collaboration to build dominance.
5. Main insights and open problems

1. The structure of networks

- A network describes a collection of nodes and the links between them.
- Once you begin to study networks it is difficult not to see them everywhere.
- Examples include: Internet, World wide web, airline networks, friendships, research alliances, co-authorships, trade and exchange, *guanxi*.

Local network of J. Tirole in 1990's



Note: Some economists might appear twice or are missing due to the use of different initials or misspellings in EconLit.

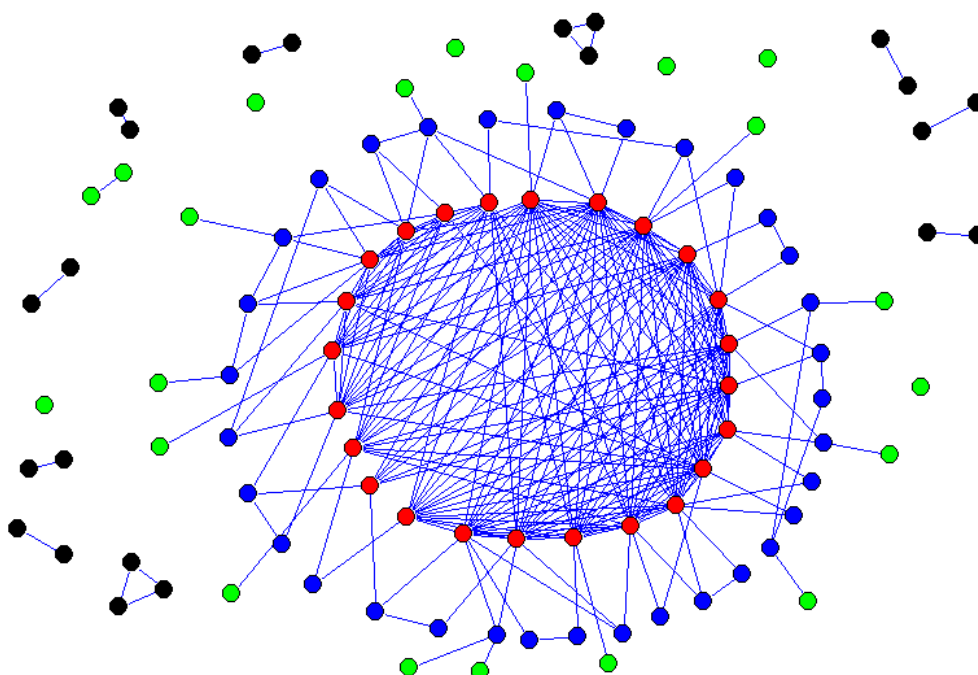


Figure 2.10: Research collaboration among firms

1. Structure of networks

- Three key properties: the **degree distribution**, the **clustering**, the **average distance**.
- **Degree distribution:** *Average degree is very small compared to number of nodes and there is enormous inequality.* On the web, in a study from 2000, there were over 200 million web sites, average degree only 7.5, most web sites less than 10 links, but some have thousands of links!
- **Clustering:** *Clustering is very high in social networks.* If links are formed at random then in a large n -node network with average degree k the clustering would be roughly k/n . In the economics co-author network the clustering coefficient was 0.157; over 7,000 times the level in a random network.
- **Average distances:** *The average distance between nodes is very small.* In the world wide web, the giant component comprised about 180 million web sites and the average distance was only 6. Similarly, in the firm network there are over 4000 nodes, but the average distance was around 4.

1. Structure of networks

- **Small worlds:** A network with small average degrees, high clustering, and small average distances has been called a *small world* network by Watts and Strogatz (1998). The expression is originally due to Milgram (1967).
- Social and economic networks display common features: low average degree, very unequal degree distribution, clustering is high and the average distance between nodes is small. Thus star network and its variants -- such as inter-linked stars and core-periphery networks -- capture essential elements of empirical networks.
- **Key questions:** Who forms the networks? When do they have this structure? Why does it matter?

2. Strategic foundations of networks

Key features of linking activity

- 1. Linking is a decision:** Individuals choose on forming links.
Examples: Scientists decide on whether or not to collaborate
Firms can choose whether or not to form an alliance;
I decide whether or not to form a hyperlink with your homepage.
 - 2. Externality/spillover:** A link between 1 and 2 affects the payoffs of 3 as well as the rewards to 3 from linking with 1 and 2.
Examples: capacity constraints in scientific collaboration;
firm 1 and 2 collaborate affects firm 3 in same market.
- Combine 1 & 2: **Games of Network Formation.**

2. Strategic foundations of networks

- Key issues in modelling:
 1. Payoffs: linking generates rewards and entails costs.
We define these formally.
 2. Power: who decides on the link, one person, two persons, all players etc.
 3. Information: what do I know -- about other players and about the network -- when I form a link?

We start with the simplest case: a player decides on whether to link with others unilaterally. No transfers or bargaining. Full Information about rewards and costs of linking and about the network.

3. Theory 1: communication networks

- **Players:** Large number, n .
- **Strategy:** Link with any subset of others. A link is 0-1.
- **Payoffs:** Each link is costly and link between 1 and 2 gives 1 access to information which 2 has on her own, **AND** information which she accesses via her links.

Thus: payoff is **increasing** in the number of people she accesses directly or indirectly and **decreasing** in the costs of links she forms.

Example: Payoff = [#people accessed] Value per person
– [cost link]. [# links formed]

- **What is the network that emerges from this process?**

3. Communication networks

- How do we solve this game?
- ***Nash equilibrium:*** This is a profile of strategies one for each player with the following property: *every player is doing as well as possible, given what others are doing.*
- What is a Nash equilibrium of the game of network formation?

3. Communication networks

Some simple intuitions

1. Suppose $C > (n-1)V$: then no linking always.

The empty network is only possibility.

2. Suppose $C < V$: then 1 wants to access everyone else irrespective of what others do.

Every player accesses everyone else; connected network is only possible outcome.

3. Suppose $V < C < (n-1)V$ then linking depends on other's linking behaviour:

If no one links, it is optimal to form no links: empty network is an equilibrium.

If people form links then it may be optimal to connected with them. This is the interesting case

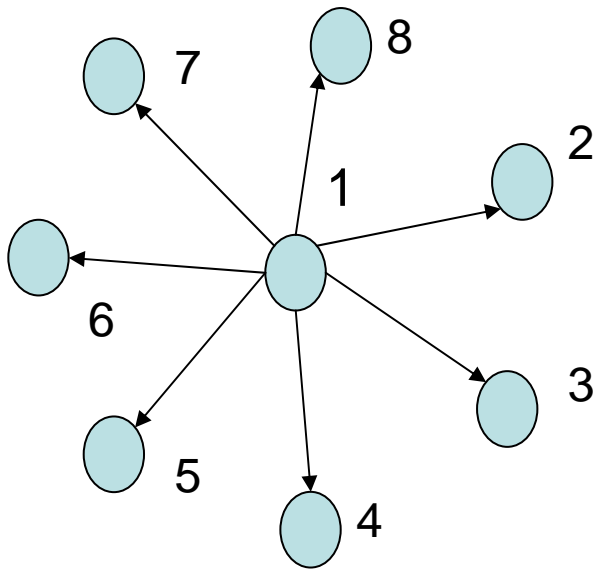
3. Communication networks

- **Theorem 1**

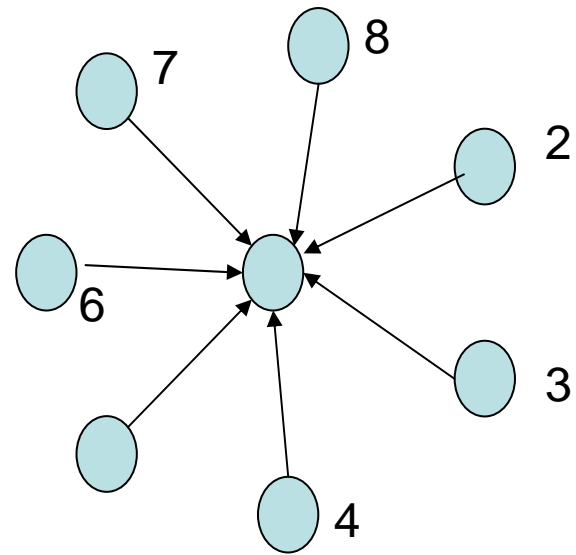
The star is the unique equilibrium network architecture.
[Bala and Goyal, 2000. Hojman and Szeidl, 2007, Ferri, 2007].

- Key intuitions:

1. *The star is an equilibrium:* the spokes are accessing everyone with just one link, and everyone is close by.
2. *Why is nothing else equilibrium?* **Networks must exhibit small world property.** Take any two end-players in a `tree' network. They have an incentive to get closer to the centre, if long paths are bad for information transmission.



Centre sponsored star



Periphery sponsored star

Equilibrium networks

3. Communication networks

- Dynamics: suppose players can observe the network and revise links over time. So network will evolve over time.
Question: starting from an arbitrary network, will the dynamics converge, and if yes, what is the long run network?
- **Theorem 2**
Starting from any network, the dynamics converge to the star network. [Bala and Goyal, 2000; Ferri 2007].

General message: unequal degrees and short average distances arise in equilibrium.

3. Communication networks: advantages of networking

- Does network degree and location confer advantages?
- Network formation leads to star in which the central hub player has privileged access to information. In general, the spokes pay for the links, and so hub gains both ways.

General message: *strategic networking can create large inequalities across players who are ex-ante identical.*

3. Communication networks: good and bad networks

- Key idea: links are motivated by individual incentives. Individual linking generates externalities and spillovers on others. So there is a tension between equilibrium and socially desirable networks.
- In communication network game: individual linking creates *positive* spillovers for others, and so individuals typically form less links than is socially desirable.

General message: *Equilibrium networks are under-connected, relative to what is socially desirable.*

4. Networks and Markets

Leading firms in hi-tech industries rely on a combination of in-house and collaborative research to stay competitive in markets.

E.g., Bayer, Merck, Pfizer etc. IT alliances

Key ideas:

- Strategic alliances in the face of competition.
 - an alliance improves competitive position of partners
 - it also alters incentives of other firms to initiate costly new alliances.

4. Networks and markets

Firms bilaterally choose research links.

- Partners share technological information which lower costs of production. More links firms lead to lower costs, which leads to larger market share.
- However each link involves a fixed cost C .

Key features: 1. link decided bilaterally.

2. alliances arise in response to market pressures and they in turn define competition in networks.

4. Networks and markets

A Game of Network Formation:

- Players: There are n firms.
- Strategies: Each firm announces intention to form 0-1 links with others. A link is formed between 1 and 2 if both 1 and 2 announce an intention to form a link.
- Payoffs: Links which are formed cost F to each firm and lower their costs of production by c . Links formed define a network, which defines a vector of firm costs.

The gains from links depend on the nature of market competition. We explore this key relation between market conditions and incentives to form alliances.

Strong competition: firm makes profits only if it is unique lowest cost.

Moderate competition: lower costs imply higher profits.

4. Networks and markets

- How do we solve games with bilateral link formation?
- Nash equilibrium is too permissive, as there is a coordination problem in bilateral link formation, firm 1 offers no link since it expects no one else to form any links.
- To avoid this problem: refine Nash equilibrium with the requirement that no two players should have an incentive to form an additional link.
- Leads to ***pairwise equilibrium***

4. Networks and markets

- **Theorem 3:**

With strong competition, the empty network is the unique pairwise equilibrium. [Goyal and Joshi (2003)]

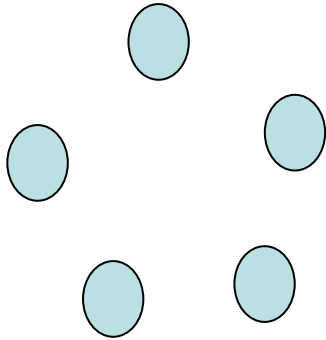
Intuition: in non-empty network, there is always a firm which forms link but makes no profits. Hence, cannot recover costs of link. Better to delete all links!

- **Theorem 4:**

With moderate competition and small costs of forming links, the complete network is the unique pairwise equilibrium. [Goyal and Joshi (2003)]

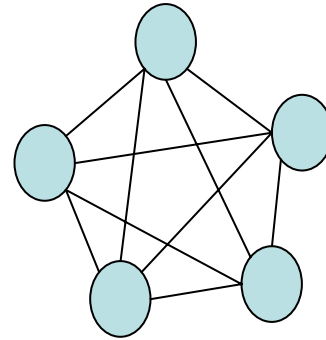
Intuition: when two firms form links, they gain at the expense of other firms, and this covers costs of links. Always form links.

General Message: market competition crucial in defining networks



Empty network

Strong competition



Complete network

Moderate competition

**Equilibrium networks under
different market conditions**

4. Networks and markets

- We now turn to the case of high costs of forming links.

FIRST NOTE: Result for strong competition unaffected.

But moderate competition now becomes very interesting:

- Key issue: additional link creates a cost of F : so link formation depends on how rewards from additional links are affected by # own and LINKS and others links:
 - Whether marginal returns are increasing/decreasing in own links?
 - Whether linking by others increases or decreases my returns?

Key property of the moderate model: **marginal payoffs are increasing in own links and decreasing in the links of other firms.**

- What are the pairwise nash equilibrium networks?

4. Networks and markets

- **Theorem 5**

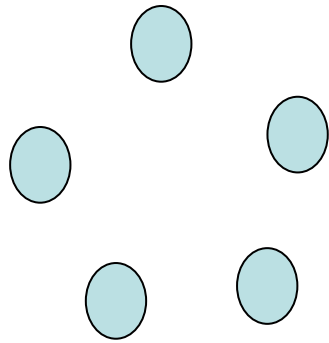
Under moderate competition with significant costs of forming links, pairwise equilibrium network has the dominant group architecture. [Goyal and Joshi (2003), Deorian (2006)]

Dominant group network contains a completely linked group of firms, and the rest of firms are isolated.

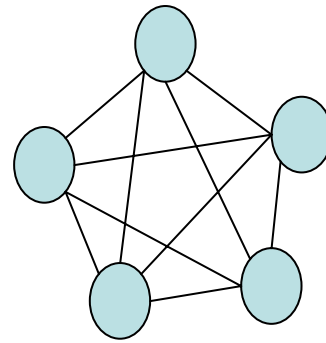
Intuition: Two or more groups of connected firms not possible, due to linear costs of links and convex returns from own links. Note that firm with connections always wants more links, but isolated firm may not link since its marginal returns are too low.

- **Theorem 6**

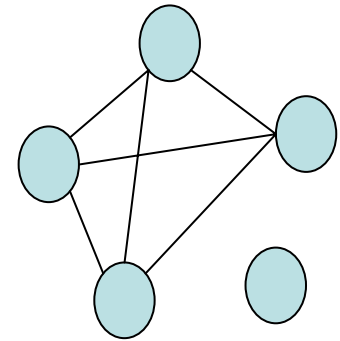
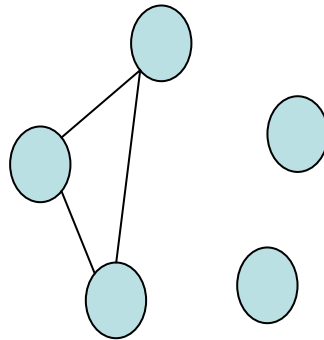
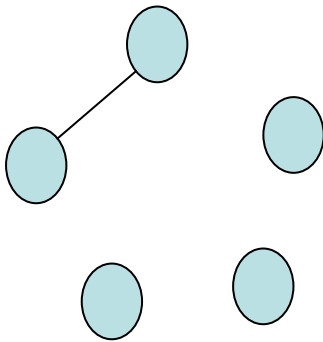
The firms in the dominant group earn strictly more profits than the isolated firms.



Empty network

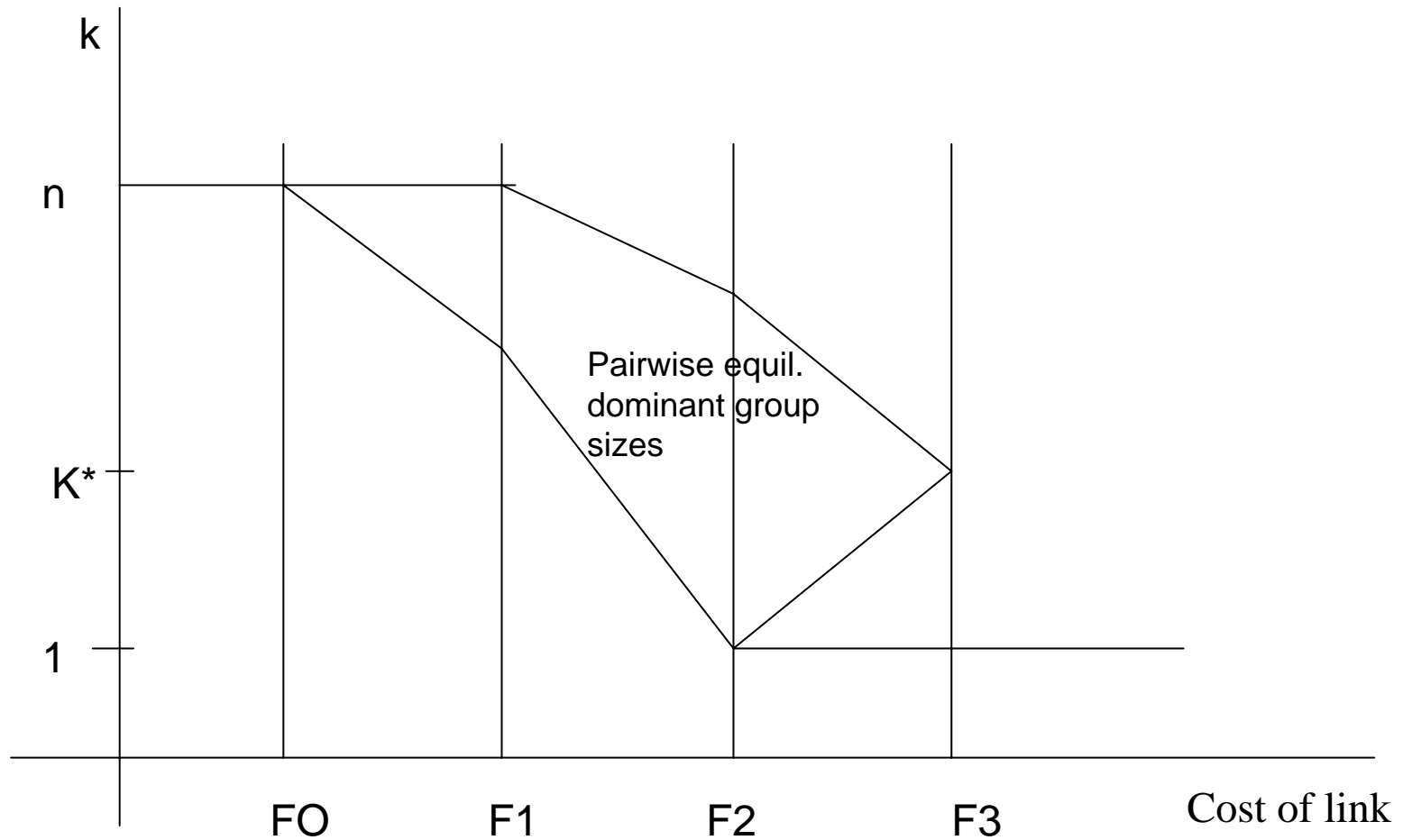


Complete network



Dominant group networks

Size of dominant group



Size of dominant group vs costs of linking

4. Networks and markets

- **Theorem 7**

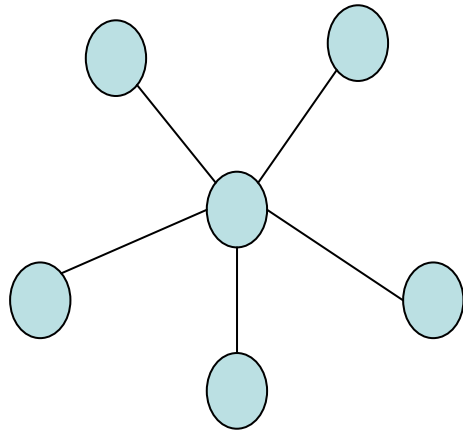
If firms can make transfers to other firms in forming links, then a star network and related structures are pairwise equilibrium networks.[Goyal and Joshi, 2003]

Intuition: Marginal gains from links are increasing in # of own links, so the central firm in a star has high marginal returns from new link. Its many connections lower the returns to peripheral firms, who wish to form no links. So central firm subsidizes links and peripheral firms form no links among themselves!

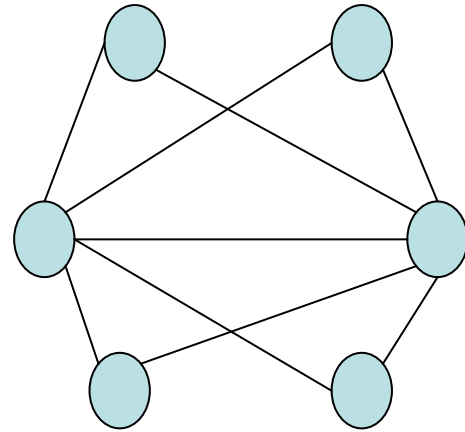
- **Theorem 8**

The central firm with more links earns larger profits than the peripheral firms with few links. [Goyal and Joshi, 2003]

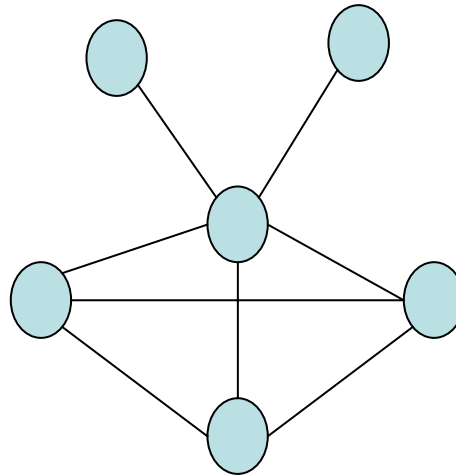
Key idea: Subsidizing links to create market advantages....



A. Star Network



B. Inter-linked star (2 centres)



C. Generalized inter-linked star

Equilibrium networks with transfers

4. Networks and markets

- General Message:** strategic networking by firms gives rise to
- unequal degrees,
 - small average distance,
 - high clustering.
 - firms with many connections actually subsidize formation of links to create larger profits [network advantages].

4. Networks and markets: socially desirable networks?

- In the firms research networks: link between 1 and 2 actually lowers profit effects of other firms; so there is a negative externality created by links: *firms create too many links in equilibrium*. [Goyal and Joshi 2003. Yi, 1998]
- Policy: Governments all over the world try and facilitate inter-firm collaborations; even subsidize them. Our results raise doubts about the optimality of such a policy.

5. Summary: strategic network formation

- The theory of network formation is concerned with understanding how networks arise out of strategic choices of players concerning link formation.
- The theory generates surprisingly sharp predictions on equilibrium networks: unequal degrees, small average distance, arise naturally. Good match with empirics.
- Strategic networking has powerful effects on payoff inequality as well as aggregate social welfare.
- Suggests role of policy – taxes and subsidies – to reorient network formation. [Taken up by the theory of mechanism design.]

5. Current research & open problems

1. Weighted graphs: Bloch & Dutta, 2008; Goyal 2005; Bruckner 2005.
2. Applications: trade, labour, crime, industry, education: Calvo & Zenou 2005; Goyal & Joshi 2006; Belleflamme & Bloch 2005.
3. Combining linking and search for jobs and prices. Galeotti 2008.
4. Linking activity and coordination and cooperation games: Jackson & Watts 2002; Goyal & Vega-Redondo, 2006; Vega-Redondo 2006.
5. Networks and markets: social networks & markets shape exchange production & consumption Kranton & Minehart 2001.
6. Pure theory of network formation: decision power, bargaining, auctions in networks. Jackson 2006; Bloch & Jackson 2006; Kranton & Minehart 2001.
7. Strategic network design: designer faced with adversary, Baccara and Bar-Isaac 2008; Goyal and Viger 2008.

5. Open problems

- A. Dynamic network formation: network advantages suggest the pressure to pre-empt others in the creation of links. Very important open problem!
- B. Network formation with large number of players: key role of incomplete information about players and about networks.
- C. Networks and markets: traditionally economists focused on markets and ignored social structures; recent work focuses on networks and ignores markets. There is an urgent need for research and models which integrate networks and markets. This is also key aspect of developing economies, as networks evolve and give way to markets in such economies.

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